The ground state of a spin- 1/2 neutral particle with anomalous magnetic moment in a two-dimensional electrostatic field

1993 J. Phys. A: Math. Gen. 26 5631

(http://iopscience.iop.org/0305-4470/26/20/043)

J. Phys. A: Math. Gen. 26 (1993) 5631–5634. Printed in the UK

COMMENT

The ground state of a spin-$\frac{1}{2}$ neutral particle with anamolous magnetic moment in a two-dimensional electrostatic field

B Roy

Physics and Applied Mathematics Unit, Indian Statistical Institute, Calcutta 700035, India

Received 18 January 1993

Abstract. We discuss the supersymmetric structure of the Aharonov–Casher effect.

It was shown by Aharonov and Casher [1] that, as in a charged particle system (AB effect [2]), topological effects can also be observed in a neutral particle system, and this is generally called the Aharonov–Casher (AC) effect. The AB and AC effects are closely related to anyons [3] and they have been studied from various points of view [4]. In particular, symmetry aspects of anionic and AB system have been studied [5–8]. Also, in some recent works [9, 10] it has been pointed out that the Direc equation with anamolous magnetic moment is related to supersymmetry. Here we shall examine, in detail, the relation between the Aharonov–Casher effect and supersymmetry. In particular, we shall investigate under what conditions the AC effect can be cast within the framework of SUSY, the phases acquired by the supersymmetric partners etc.

We recall that the Dirac equation with an anamolous magnetic moment μ reads

$$(\partial + \tfrac{1}{2}\mu\sigma_{\gamma\delta}F^{\gamma\delta} + m) = 0 \tag{1}$$

where $F^{\gamma\delta}$ is the field strength and m is the mass of the particle. In the case of the AC effect the electric field is such that there are regions of space where $\nabla\cdot\boldsymbol{E}=0$ and there are also some regions where $\nabla\cdot\boldsymbol{E}\neq 0$. The specific conditions for the wavefunction to acquire the topological AC phase are [3, 11]:

(i) a neutral spin-$\frac{1}{2}$ particle with a non-zero anamolous magnetic moment is moving on a plane in an external electric field $\boldsymbol{E}$;
(ii) on the plane, $E_3=0$, $\partial_3 E_3=0$ and $\partial_3\psi=0$;
(iii) it is possible for the particle to traverse a closed path in the region where $\nabla\cdot\boldsymbol{E}=0$ and is such that a surface whose boundary is the path intersect regions where $\nabla\cdot\boldsymbol{E}\neq 0$.

Now, writing the wavefunction as

$$\psi = \begin{pmatrix}\varphi\\ \chi\end{pmatrix} \tag{2}$$

where φ and χ are two-component spinors, equation (1) can be written as

$$\frac{1}{2m}\boldsymbol{\sigma}\cdot(\boldsymbol{p}-\mathrm{i}\mu\boldsymbol{E})\boldsymbol{\sigma}\cdot(\boldsymbol{p}+\mathrm{i}\mu\boldsymbol{E})\varphi = \frac{1}{2m}(\varepsilon^2-m^2)\varphi \tag{3}$$

$$\frac{1}{2m}\boldsymbol{\sigma}\cdot(\boldsymbol{p}+\mathrm{i}\mu\boldsymbol{E})\boldsymbol{\sigma}\cdot(\boldsymbol{p}-\mathrm{i}\mu\boldsymbol{E})\chi = \frac{1}{2m}(\varepsilon^2-m^2)\chi \tag{4}$$

0305-4470/93/205631+04 \$07.50

where $\boldsymbol{\sigma}=(\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices and ε denotes the energy.

If the electric field satisfies conditions (i) and (ii) then equations (3) and (4) can be written as

$$\frac{1}{2m}[(p_1-\mu E_2\sigma_3)^2+(p_2+\mu E_1\sigma_3)^2+\mu(\nabla\cdot\boldsymbol{E})]\varphi=\frac{1}{2m}(\varepsilon^2-m^2)\varphi \tag{5}$$

$$\frac{1}{2m}[(p_1+\mu E_2\sigma_3)^2+(p_2-\mu E_1\sigma_3)^2-\mu(\nabla\cdot\boldsymbol{E})]\chi=\frac{1}{2m}(\varepsilon^2-m^2)\chi. \tag{6}$$

We note that for equations (5) and (6) there is a gauge symmetry and the electric field becomes a pure gauge vector potential. It can be easily seen that if we take

$$\begin{aligned} \boldsymbol{A}&=\mu\sigma_3(E_2, -E_1, 0)\\ \boldsymbol{A}'&=-\mu\sigma_3(E_2, -E_1, 0) \end{aligned} \tag{7}$$

then

$$\begin{aligned} \nabla\times\boldsymbol{A}&=-\mu\sigma_3(\nabla\cdot\boldsymbol{E})\hat{k}\\ \nabla\times\boldsymbol{A}'&=\mu\sigma_3(\nabla\cdot\boldsymbol{E})\hat{k} \end{aligned} \tag{8}$$

and in the charge-free region, equations (5) and (6) become

$$\begin{aligned} \frac{1}{2m}(p_1^2+p_2^2)\varphi'&=\frac{1}{2m}(\varepsilon^2-m^2)\varphi'\\ \frac{1}{2m}(p_1^2+p_2^2)\chi'&=\frac{1}{2m}(\varepsilon^2-m^2)\chi' \end{aligned} \tag{9}$$

where

$$\begin{aligned} \varphi'&=\exp\left[-i\mu\sigma_3\int \boldsymbol{E}\times\hat{k}\,\mathrm{d}r\right]\varphi\\ \chi'&=\exp\left[i\mu\sigma_3\int \boldsymbol{E}\times\hat{k}\,\mathrm{d}r\right]\chi. \end{aligned} \tag{10}$$

The topological phases for the two components are then given by

$$\alpha=-\mu\sigma_3\oint(\boldsymbol{E}\times\hat{k})\mathrm{d}r=\mu\sigma_3\int_S(\nabla\cdot\boldsymbol{E})\mathrm{d}S=\mu\sigma_3\lambda \tag{11}$$

$$\alpha'=\mu\sigma_3\oint(\boldsymbol{E}\times\hat{k})\mathrm{d}r=-\mu\sigma_3\int_S(\nabla\cdot\boldsymbol{E})\mathrm{d}S=-\mu\sigma_3\lambda \tag{12}$$

where λ is the linear charge density.

To see the role of supersymmetry we note that each of equations (5) and (6) has a two-component structure and the Hamiltonians corresponding to these equations are

$$\begin{aligned} H_1&=(p_1-\mu E_2)^2+(p_2+\mu E_1)^2+\mu(\nabla\cdot\boldsymbol{E})\\ H_2&=(p_1+\mu E_2)^2+(p_2-\mu E_1)^2+\mu(\nabla\cdot\boldsymbol{E})\\ H_3&=(p_1+\mu E_2)^2+(p_2-\mu E_1)^2-\mu(\nabla\cdot\boldsymbol{E})\\ H_4&=(p_1-\mu E_2)^2+(p_2-\mu E_1)^2-\mu(\nabla\cdot\boldsymbol{E}). \end{aligned} \tag{13}$$

Let us now turn to non-relativistic SUSYQM. If we define [12] two supercharges Q^1 and Q^2 by

$$\left.\begin{aligned}&\{Q^a, Q^b\}=2H\delta^{ab}\\&[Q^a, H]=0\end{aligned}\right\} a, b=1, 2 \tag{14, 15}$$

then the $N=2$ SUSY Hamiltonian is given by

$$2H=(p_1+A_1)^2+(p_2+A_2)^2+(\nabla\times A)_z\sigma_z. \tag{16}$$

Clearly, if we take

$$\begin{aligned}A_1&=-\mu E_2\\A_2&=\mu E_1\end{aligned} \tag{17}$$

then from (8) and (13) it follows that H_1 and H_4 are supersymmetric partners. Similarly H_2 and H_3 are supersymmetric partners. Thus the system represented by (13) is equivalent to two non-relativistic $N=2$ supersymmetric quantum mechanical systems. Also, from equations (11) and (12), it follows that the topological phases acquired by the SUSY partners are the same, i.e. for H_1 and H_4 it is $\mu\lambda$ while for H_2 and H_3 it is $-\mu\lambda$.

Let us now consider a specific example and take

$$\begin{aligned}A_1&=-\mu E_2=-\frac{\mu x_2}{r^2}\\A_2&=\mu E_1=\frac{\mu x_1}{r^2}\\r^2&=x_1^2+x_2^2.\end{aligned} \tag{18}$$

Clearly we have

$$\nabla\times \boldsymbol{A}=\nabla\cdot \boldsymbol{E}=\delta(\boldsymbol{r}) \tag{19}$$

and thus all the conditions for the AC effect are satisfied. However, it can be shown [7, 13] that if A_1, A_2 is of the form (18) then H_1, H_4 and H_2, H_3 are supersymmetric partners everywhere except the origin (i.e. spectral degeneracy holds between the components).

Let us now consider equation (1) in the non-relativistic limit. It is pointed out that when we speak of non-relativistic limit, we consider the four-component Dirac equation as a whole, and upon taking the limit only the upper two components (i.e. φ) of the wavefunction ψ survive and the resulting Hamiltonian is given by

$$H_{NR}=\frac{1}{2m}(\boldsymbol{p}-\mu\boldsymbol{E}\times\boldsymbol{\sigma})^2. \tag{20}$$

We note that it is inappropriate to take the non-relativistic limit of Q^1 and Q^2 because Q^1, Q^2 and H constitute a non-relativistic SUSYQM system.

Now the Hamiltonian in (20) does not represent a SUSY Hamiltonian. This follows from (13) as well as from the fact that spin is not coupled to the electric field. This is in contrast to the AB effect where the magnetic field is coupled to the spin.

In conclusion, we have discussed the underlying SUSY structure of the AC effect. In particular it has been shown that the AC effect can be formulated within the framework of $N=2$ non-relativistic supersymmetric quantum mechanics. The AC phases of different supersymmetric partners have also been found.

References

[1] Aharonov Y and Casher A 1984 *Phys. Rev. Lett.* **53** 319
[2] Aharonov Y and Bohm A 1959 *Phys. Rev.* **115** 485
[3] Hagen C R 1991 *Int. J. Mod. Phys.* A **6** 3119
[4] Gerbert Ph 1991 *Int. J. Mod. Phys.* A **6** 173
[5] Jackiw R 1990 *Ann. Phys.* **201** 83
[6] Roy B, Barut A O and Roy P 1993 *Phys. Lett.* **172A** 316
[7] Roy P and Tarrach R 1992 *Phys. Lett.* **274B** 59
[8] Allen T J 1991 *Nucl. Phys.* B **360** 409
[9] Semenov V V 1990 *J. Phys. A: Math. Gen.* **23** L721
[10] Semenov V V 1992 *J. Phys. A: Math. Gen.* **25** L619
[11] He X G and McKellar B H J 1991 *Phys. Lett.* **264B** 129
[12] de Crombrugghe M and Rittenberg V 1983 *Ann. Phys.* **151** 99
[13] Park C H 1992 *Nucl. Phys.* B **376** 99